SF3961 Statistical Inference

2015/16

Homework 6

This is preparation for the discussion on classical and Bayesian statistics. Think through the following questions and write down (and hand in) brief answers (no essays). The main reference is David Mackay's book "Information theory, Inference, and Learning Algorithms" available at

`http://www.inference.phy.cam.ac.uk/itprnn/book.pdf`

Further references are given below.

Section 37.1 in Mackay

- What arguments do Mackay present against the classical (sampling theory) approach?
- Does he make a fair evaluation of the classical approach?
- What criticism can you find agains Mackay's Bayesian solution?

Section 37.2 in Mackay

- Is Mackay's calculations correct?
- Does he make a fair evaluation of the classical approach?
- What section in Lindley's article is related to the discussion in Section 37.2?
- At the end of Section 37.2 Mackay discusses an example given in Exercise 35.4, p. 446 (and his solution on p. 449). Look carefully at this example, you will also find the references (Kepler and Oprea, 2001) and (Luria and Delbrück, 1943) below. Does this example provide valid evidence against the classical approach?
- Study carefully the model by Luria and Delbrück for the number of resistant bacteria. It is described in their section labelled "THEORY". Show, using modern probabilistic reasoning, that the number of resistant bacteria in a culture with N_t bacteria can be represented as

$$\int_0^t e^{t-s}\Lambda(ds),$$

  where Λ is an inhomogeneous Poisson process with intensity function $\lambda(t) = aN_t$. That is, Λ has independent increments with $\Lambda(t) - \Lambda(s)$ having Poisson distribution with parameter $\int_s^t \lambda(u)du$.
- A challenge, if you are up for it, is then to relate the above expression for the number of

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resistant bacteria to Mackay's solution of Exercise 35.4 by showing that

$$\int_0^t e^{t-s}\Lambda(ds) =^{\mathrm{d}} \sum_{i=1}^{M} \xi_i,$$

where $=^{\mathrm{d}}$ denotes equality in distribution, M has Poisson distribution with parameter aN_t, $\xi_1, \xi_2, \ldots$ are iid with standard Pareto distribution, $P(\xi_i \leq x) = 1 - x^{-1}$, $x \geq 1$, and M is independent of $\xi_1, \xi_2, \ldots$

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Other references

1. Luria, S.E. and Delbrück, M. (1943): Mutations of bacteria from virus sensitivity to virus resistance, *Genetics*, Vol. 28.
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4. Kepler, T.B. and Oprea, M. (2001): Improved inference of mutation rates *Theoretical Population Biology*, Vol. 59, 41-48.
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